<u>Algebraic Number Theory exam remarks</u>

This is a summary of remarks that I made during lectures and in conversations with some of you.

- The format of the exam will be 4 out of 5 questions; all carry questions equal weight.

<u>Regarding past papers:</u> Any past course called “Algebraic Number Theory” or “Local Fields” might contain problems that one can practice on. I remind you, however, that the syllabus can and have changed from year to year, so not all questions may be suitable. As far as I know, the 2015-16 course on Algebraic Number Theory (examined in 2016) is the one with the closest syllabus.

<u>Regarding exam structure:</u> The exam will be roughly 50/50 bookwork/problems. It is designed to test your understanding of the core content of the course, and the problems are in the spirit of the example sheets (in particular sheets 2-4), so I expect you to revise the example sheets as well as the lecture notes.

By the core content of the course, I mean the following topics (broadly interpreted):

- The basic theory of local fields: Hensel’s Lemma, unramified/totally ramified extensions, Newton polygons…
- Ramification theory: Decomposition groups, inertia groups and higher ramification groups.
- Places of number fields, the equivalence between finite places and primes, definition of adeles and ideles.
- The splitting behaviour of primes in finite extensions of number fields, in particular the relation between this and the previous two items.
- Statements of class field theory (local and global), explicit class field theory for Q and Qp (cyclotomic fields), the relation between the Hilbert class field and the class group.

Finally, some scattered remarks that I made at various points:

- While I expect a “working knowledge” of all parts of the course, proofs on the initial commutative algebra results (localisation, Nakayama etc.) or the infinite Galois theory results are non-examinable. This also applies to the corresponding questions on the example sheets.

- You will not be asked to compute class numbers via methods such as the Minkowski bound.

- The bookwork questions are intended to be the kinds of questions that I would expect someone with a very good understanding of the course to be able complete with minimal preparation. As a non-example of this, while I expect you to know the upper numbering of ramification groups and that this numbering behaves well with respect to quotients of Galois groups, I won’t examine the proof of this.

- This is perhaps obvious, but you are certainly not expected to know proofs of results that were not proved in the course.

- There will be no essay question for this course.